Dynamic Vibration Absorbers – Application with Variable Speed Machines

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If a resonance condition occurs in the field, one effective and low cost method to alleviate it is to install a dynamic vibration absorber. This discussion explains how a dynamic vibration absorber can suppress vibration over a wide operating speed range.

With the wide use of variable frequency drives in the pump industry and increasing unit size, it is becoming more difficult to design mechanical systems free from natural frequencies within operating speed range. If such an occurrence is allowed in the field, a resulting resonance condition threatens to significantly impact performance and longevity of the equipment.

Traditional treatment methods that involve structural modifications are often time consuming and expensive. Blocking the problem frequencies in the variable frequency drive limits the use of the system by the user. One possible solution is an installation of a *dynamic vibration absorber* (DVA).

The DVA has certain advantages over other methods of vibration suppression. It is external to the machine structure, so no re-installation of equipment is necessary. Unlike with structural modifications, when the final effect is unknown until mass-elastic properties of the machine components have been modified, a DVA can be designed and tested before installation. It can be adjusted in the lab environment with predictable field results.

In many scenarios, this offers an economical vibration reduction solution. Figure 1 schematically shows one possible method of installation of a DVA on a pump.

History and Principle of Operation

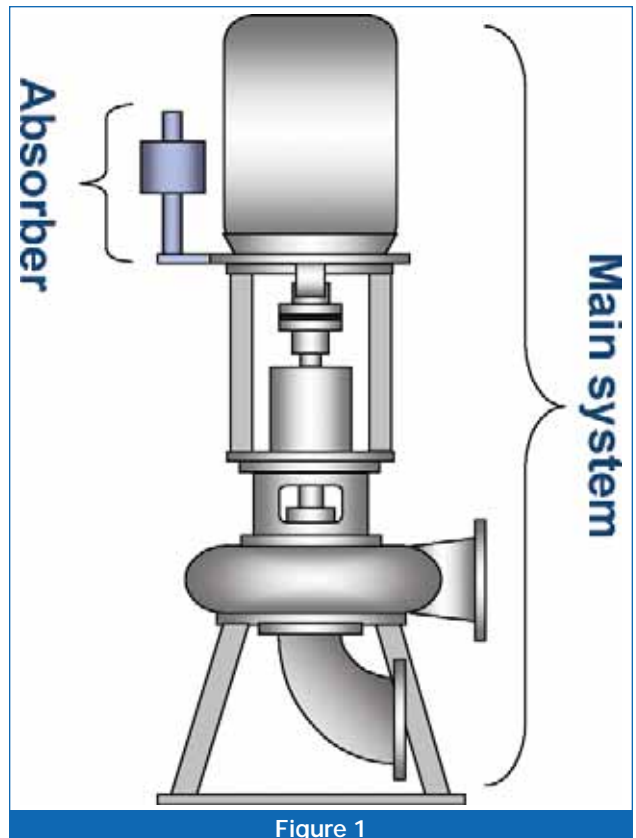The dynamic vibration absorber was invented in 1909 by

Figure 1

Hermann Frahm (US Patent #989958, issued in 1911), and since then it has been successfully used to suppress wind-induced vibration and seismic response in buildings. Characteristics of DVA were studied in depth by Den Hartog (1985).

In the industry, it has been primarily used to suppress vibration caused by a resonance condition in machinery. A DVA, sometimes referred to as a *tuned mass damper*, consists of a spring-mass system installed on a vibrating machine. In its classic form, its natural frequency is tuned to match the natural frequency of the machine it is installed on. Because of this tuning a DVA exerts a force on the main system that is equal and opposite to the excitation force, canceling vibration at the resonant frequency.

In modern applications, the goal is to assure the performance within specifications over a wide frequency range while minimizing the size of the device. A DVA is viewed by many engineers as a single frequency device. Analysis and an application example (presented below) demonstrate how vibration suppression in a wide frequency range can be achieved.

Dynamic Model

For simplicity, we will consider a dynamic model for a machine as a single degree of freedom system consisting of a single mass and a single spring. We will use a similar model for the dynamic vibration absorber. When the DVA is installed on the main system, the result is a two degree of freedom system whose dynamic model is shown in Figure 2.

In this system, the coordinate x_1 corresponds to the displacement of the main mass M, and the coordinate x_2 corresponds to the displacement of the absorber mass m. The main system's stiffness is represented by the equivalent spring
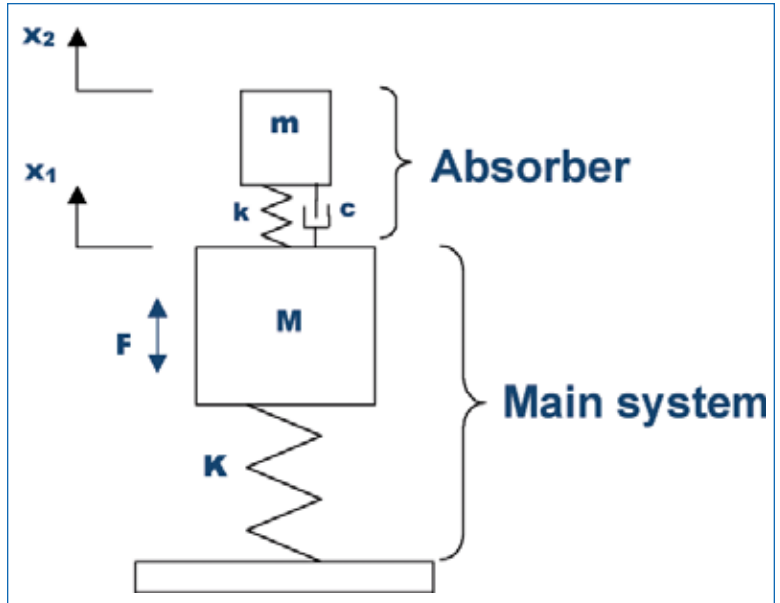

Figure 2

K, while the absorber system has the spring k. The absorber system has a viscous damping element c while the main system is considered undamped. The main system is excited by a periodic force F that in rotating machines is usually represented by residual imbalance force, but could be any periodic excitation originating in the machine, such as vane passing excitation in centrifugal pumps.

The system above is described mathematically by a system of two ordinary differential equations. Employing some standard math (Den Hartog, pp. 93-96), the formula for the system response can be derived. We will plot the results of this formula with varying absorber parameters.

First, a few variables and dimensionless ratios must be introduced, since the results will be easier to handle in this form (see Formulas 1).

Now we are ready to plot the results. First, we will evaluate the effect of an undamped dynamic absorber with the absorber tuned to the main system natural frequency, so that the tuning ratio $f = 1$ (damping ratio $\zeta = 0$). These results are shown in Figure 3.

It is notable how the dynamic absorber cancels vibration at the resonance frequency.

$\mu = \dfrac{m}{M}$ - ratio of the absorber mass to the main mass

$f = \dfrac{\omega_n}{\Omega_n}$ - ratio of the absorber natural frequency to the main system natural frequency (tuning ratio)

$x_{st} = \dfrac{F}{K}$ - static deflection of the main system under the force with magnitude F

$\zeta = \dfrac{c}{c_c}$ - ratio of the absorber damping to the critical damping (damping ratio); critical damping is defined as $c_c = 2 \cdot m \cdot \omega_n$, where m is the absorber mass and ω_n is the natural frequency of the absorber

$g = \dfrac{\omega}{\Omega_n}$ - ratio of the forcing frequency to the natural frequency of the main system (dimensionless operating speed)

$X = \dfrac{x_1}{x_{st}}$ - ratio of the main mass displacement to the static deflection (dimensionless response at the mass M, or amplification factor)
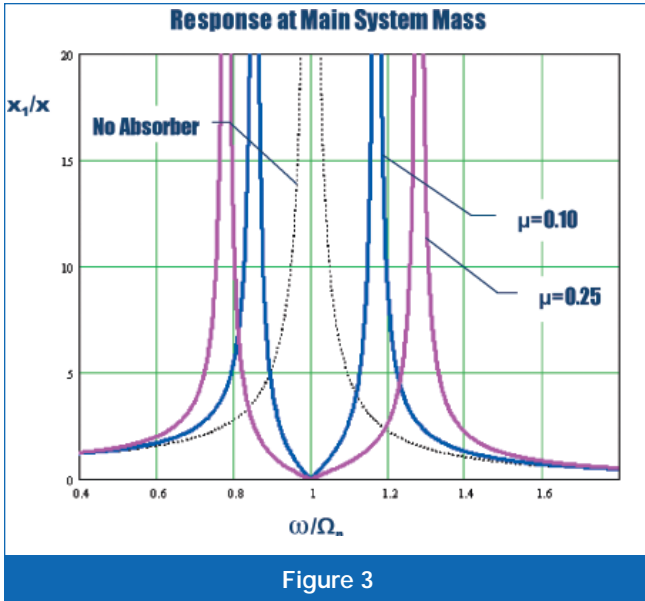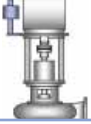
Formulas 1

Figure 3

Figure 4

Instead, it creates two new natural frequencies, one below and one above the original natural frequency. This happens because with the absorber the system has two degrees of freedom and hence two corresponding natural frequencies. The width between the two new natural frequencies depends on the mass ratio μ. Figure 3 shows the response with two different mass ratios.

With a larger absorber mass the natural frequencies sit wider apart, so a wider safe operating range around the original resonant frequency can be achieved. However, the large absorber mass very quickly becomes impractical, especially for large machinery. Figure 4 shows the two new natural frequencies in relation to the mass ratio of the absorber.

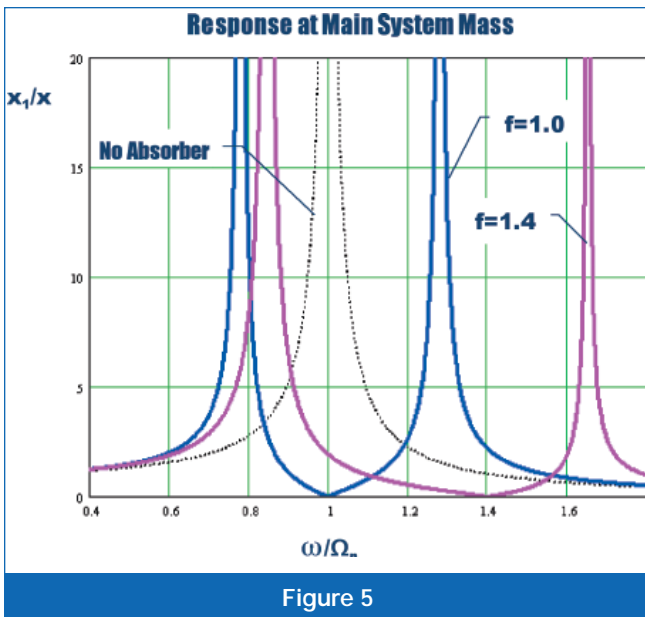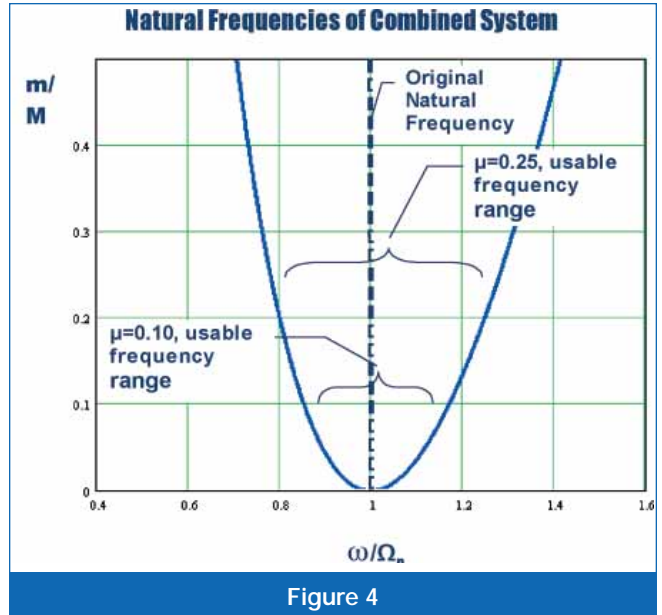By changing the tuning ratio of the absorber, the position

of the two new natural frequencies and a usable operating speed range between them can be further adjusted. Figure 5 shows the effect of tuning on the natural frequencies of the combined system with an undamped absorber (damping ratio ζ = 0).

Two curves represent two absorber systems: one with the standard tuning ratio f = 1 (blue lines), and the other one with the tuning ratio f = 1.4, representing an over tuned absorber system (magenta lines). The over tuned absorber creates a slightly higher low natural frequency, but significantly extends the range into the area of high frequencies.

Figure 6 shows the two natural frequencies of the combined system in relation to the tuning ratio. By varying tuning and mass ratios, a necessary operating speed range free of natu-
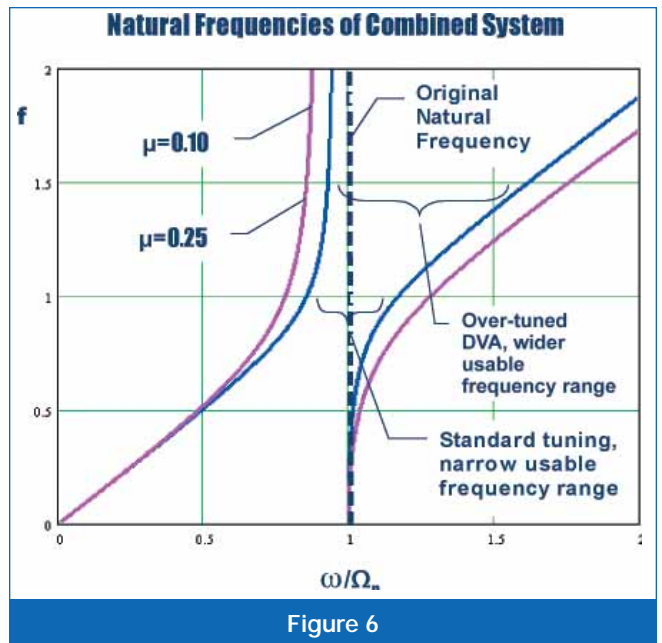

Figure 5

Figure 6

ral frequencies can be achieved with an undamped DVA.

This is important because an undamped absorber is simple to design and manufacture and its adjustment is less complicated than in a damped absorber that is described below. The tradeoff is that for a wide frequency range a required undamped absorber may become quite large.

Damped Dynamic Vibration Absorber

By adding viscous damping c to the absorber system, a better attenuation in a wide frequency range can be achieved. A relatively small amount of damping is needed to reduce vibration at the two natural frequencies.

For best effect, a damped DVA should be optimized so that its tuning and damping ratios are calculated and adjusted for a specific mass ratio (Den Hartog, pp. 100, 103). An optimized damped DVA has the same amplification factor for both natural frequencies. This amplification factor X can then be calculated for any mass ratio. An optimally tuned and damped system response is shown in Figure 7. The value of X is shown as well.

This result demonstrates that an optimized damped dynamic absorber suppresses resonance vibration within a wide frequency range. This is a universal solution as it works for any frequency range. The amplification factor is controlled by the mass ratio, so an absorber can be designed to meet a specific vibration limit.

Application Example

For variable speed pump applications, a DVA can be designed with various optimization criteria. The following example demonstrates three types of possible optimization.

The first solution uses an undamped DVA. This solution is easy to design and implement, but it requires a quite large absorber mass. The second solution is an optimally tuned and damped DVA. It is a universal solution for any operating speed range. The third solution is optimized for the smallest absorber size.

Let's assume that a pump motor in Figure 1 is operating at 1760-rpm with a 30 percent speed turn down. This defines the speed range of 1230-rpm to 1760-rpm. For the purpose of this example, let's assume that the system has a natural frequency at 1800-rpm. The mass of the motor is 500-lbs and the mass of the rotor is 200-lbs. The rotor is balanced to the grade G6.3 according to ISO-1940. The maximum allowable vibration limit is 0.25-in/s RMS.

Let's design a dynamic vibration absorber to bring the vibration of the motor within the specifications.

Solutions

Common to all three solutions is the calculation of the maximum amplification factor allowable within the operating speed
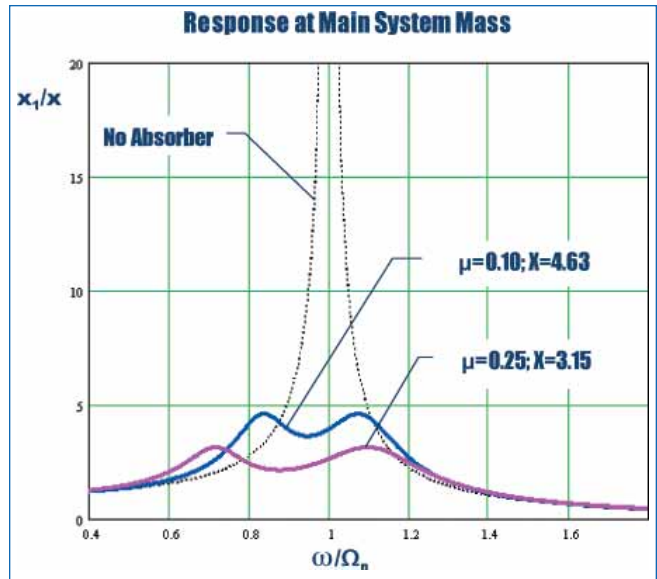

Figure 7

range. For this reason, steps 1 through 3 below apply to all three solutions.

Step 1. Find the residual imbalance force. For simplicity we assume that the residual imbalance is in single plane. The following formula can be used:

$$F = m_r \cdot G \cdot \omega$$

where:

F – centrifugal force due to residual static imbalance

m_r – mass of the rotor

ω – angular velocity of the rotor, $\omega = \dfrac{2 \cdot \pi}{60} \cdot RPM$

G = 6.3mm/s – circumference speed of the center of gravity (grade G6.3)

Converting to common units and performing calculations we find that F = 23.7 lbf

Step 2. Find the static deflection of the main mass under the excitation force. This value would be the magnitude of vibration if no structural amplification existed. To do this we will first find the equivalent system stiffness:

$$K = M \cdot \Omega_n^2$$

where:

M – motor mass

Ω_n – natural frequency of the main system

The static deflection is found as follows:

$$x_{st} = \frac{F}{K}$$

This yields x_{st} = 0.54 mils

Step 3. Find the allowable amplification factor X. To do this we first need to convert the vibration level to peak mils from RMS velocity to find the maximum allowable displacement of mass M:

$$x_1 = \sqrt{2} \cdot \frac{v\,RMS}{\omega}$$

In this formula we will use the motor speed of 1800-rpm to find ω, finding x_1 = 1.88 mils

$$X = \frac{x_1}{x_{st}}$$, as was defined above. This results in allowable amplification factor X = 3.5

Solution 1. Undamped DVA

The response plot for an undamped DVA solution is shown in Figure 8. This absorber uses the mass ratio μ = 0.25 and tuning ratio f = 0.70.
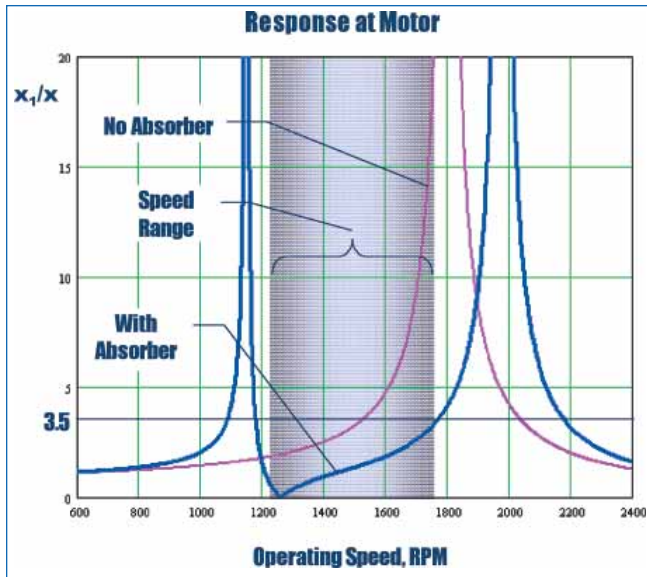

Response at Motor

Figure 8

There is no analytical formula that would help to arrive to this solution; however, Figure 6 may serve as a starting point. The final parameters are found by plotting the response curves using the formula for response at the main system mass (Den Hartog p. 96). The solution in Figure 8 includes a small margin on both sides of the speed range, so the field adjustment would be possible.

Solution 2. Damped DVA

Step 4. Find the optimized absorber parameters. The optimal tuning and damping ratios are found through the mass ratio μ, which can be found through the amplification factor X for the optimally tuned and damped absorber (Den Hartog p. 100). The following approximate for-

mulas can be used (see Formulas 2).

The formulas in Formulas 2 give a close first approximation. Analysis of the results shows that the maximum amplification factor is 3.64, slightly higher than the maximum allowable amplification factor of 3.5. After adjustment the new parameters are: mass ratio μ = 0.20, tuning ratio f = 0.84 and damping ratio ζ = 0.21. The plot is shown in Figure 9.

The importance of this result is that an absorber having a mass of 20 percent of the main system mass tuned to 84 percent of the main system natural frequency and having the damping ratio of 21 percent will bring the vibration of the motor within the specifications, regardless of the operating speed range.

Solution 3. Damped DVA Optimized for the Smallest Absorber Mass

The previous solution reduces vibration over the full frequency spectrum. Further optimization is possible if we limit the attenuation only to the frequency range in question as we did in Solution 1.

In the plot for the Solution 2, the response curve has two "humps" corresponding to two natural frequencies of the system, both under the limit of 3.5 for the amplification factor. The higher frequency "hump" is outside of the operating speed range of the motor and should not be controlled. It is also below the lowest frequency that could be excited by twice the rotating speed at 2460-cpm. By allowing higher amplification factor at this frequency, the mass of the absorber can be further reduced.

Parameters for this solution can be found only through plotting of various scenarios. The solution for mass ratio μ = 0.15, tuning ratio f = 0.80, and damping ratio ζ = 0.11 is shown in Figure 10.

Further Optimization

Excitation forces in pumps and other rotating machines are typically functions of rotating speed. Further optimization is

$$X = \sqrt{1 + \frac{2}{\mu}}$$, where μ is the mass ratio as defined in Step 4. Finding μ from here:

$$\mu = \frac{2}{X^2 - 1}$$, and calculating for X = 3.5, we obtain the minimum μ = 0.18

$$f = \frac{1}{1 + \mu}$$, the optimal tuning ratio f = 0.85

$$\zeta = \sqrt{\frac{3}{8} \cdot \frac{\mu}{(1 + \mu)^3}}$$, the optimal damping ratio ζ = 0.2
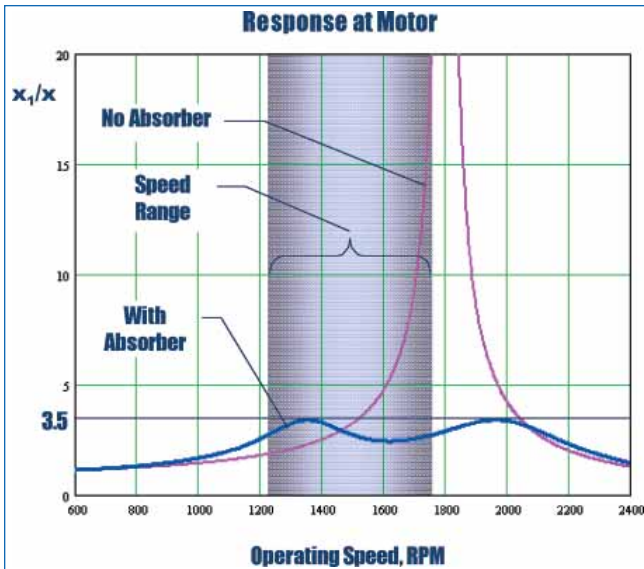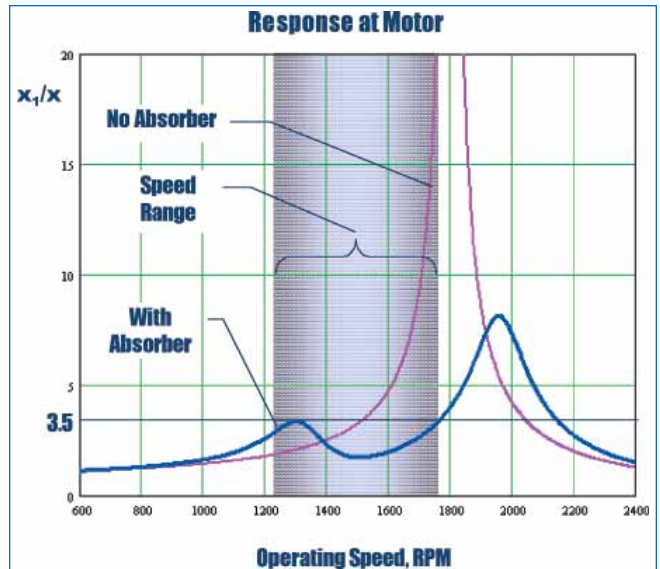
Formulas 2

Figure 9

Figure 10

possible if these relationships are known.

For example, the imbalance force is proportional to the square of rotation speed. In the application example above it was considered constant for clarity. If we consider a proper relationship for the imbalance force, the absorber mass can be further reduced because the excitation force will be lower for the lower end of the operating speed range.

Application Notes

- The curves in the above example show response in the units of displacement. Similar analysis can be done for units of velocity or acceleration.
- Real life designs may not fit exactly into the dynamic model used in this article. Additional analysis may be required.
- As vibration at the main system mass is suppressed, the absorber mass vibration is not. This should be considered in the design of the absorber spring element.
- One of the advantages of a dynamic vibration absorber is that it can be tested and fine tuned without the main system. Absorber mass, natural frequency and the damping ratio are independent of the main system and can be easily determined by a test before installation on a machine.

It should be noted that the use of DVA with machinery should not be viewed only as a fix to a vibration problem. In some cases, when a machine structure has to be tall with a high center of gravity, a DVA can be designed to be built into a machine, very much like it would be installed in a tall building.

For example, heavy motors designed for vertical installation often have their fundamental natural frequency (often called reed frequency) just slightly above the operating speed. If such a motor is installed above a pump on a pedestal, the system natural frequency can get dangerously close to the operating range, causing a resonance.

A DVA could be incorporated into the motor structure and tuned appropriately to prevent resonance vibration. For this purpose an element of the motor structure, such as a fan cover, may serve as an absorber mass if it is mounted to the motor by elastic springs.

Conclusions

Parametric analysis of the main mass response in a system equipped with a dynamic vibration absorber demonstrates that resonance vibration can be successfully controlled in such a system for a wide operating speed range.

Various optimization methods provide a range of options for applications with variable speed machines. The application example proves the feasibility of a solution involving a dynamic vibration absorber.

P&S

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